

The One-Body Approximation & The Role of the Spectroscopic Factor

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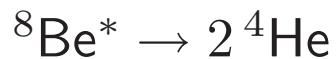
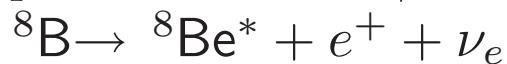
Outline

- The role of single particle approximations in radiative capture
 - Cross-Section proportional to Spectroscopic factor
- One-body approximations and proton (and alpha) emission
 - Spectroscopic factor not an observable?
- Particle-hole versus Particle-only approaches.

${}^7\text{Be}(\text{p},\gamma){}^8\text{B}$

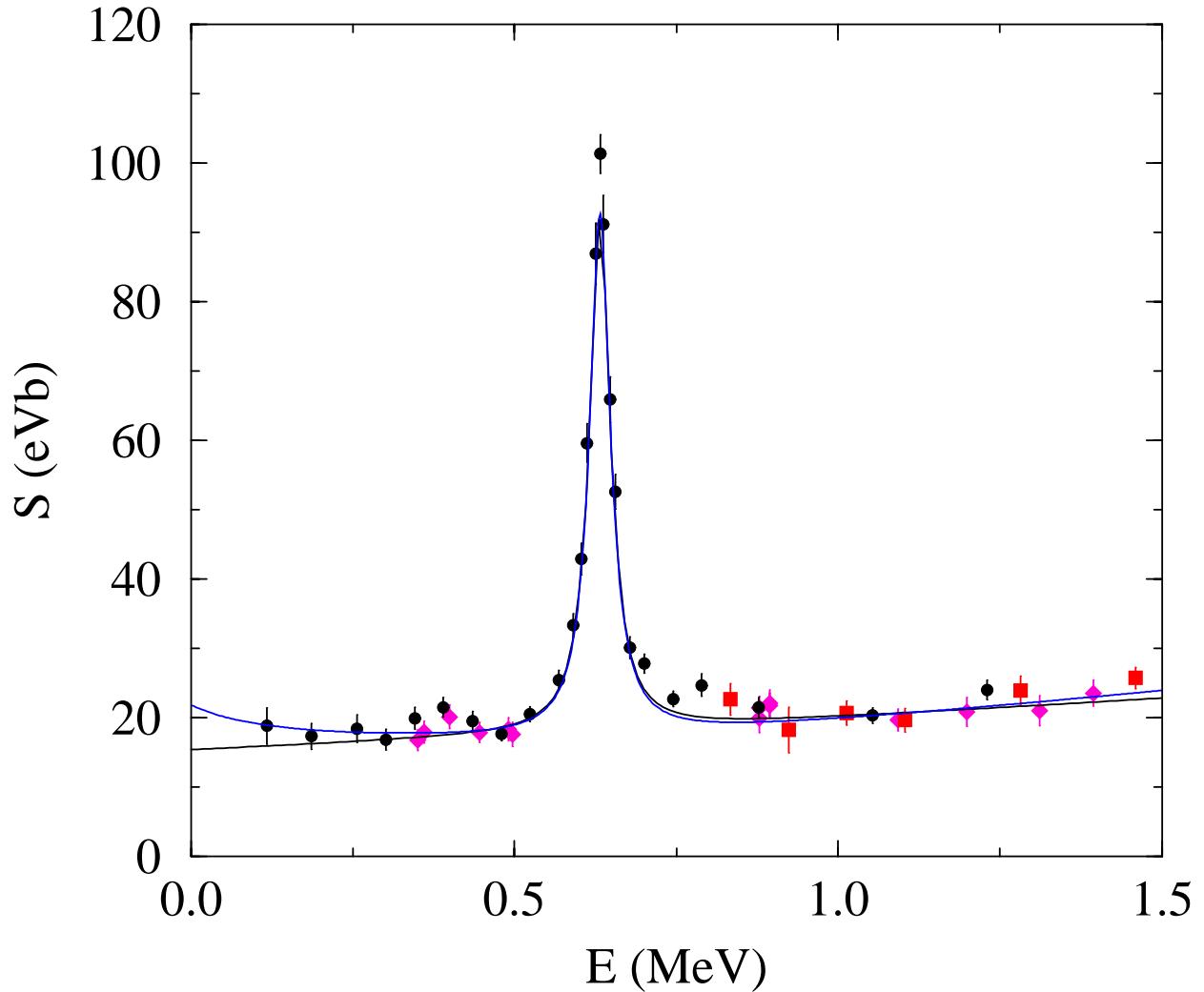
- “Simple” reaction
 - Well Studied, cluster models, potential models, etc

- Important for Solar Neutrino studies



is associated with about 0.02% of the produced ${}^4\text{He}$. This is insignificant energetically, but the resulting B neutrino spectrum extends to much higher energy than the others, so they are easier to detect.

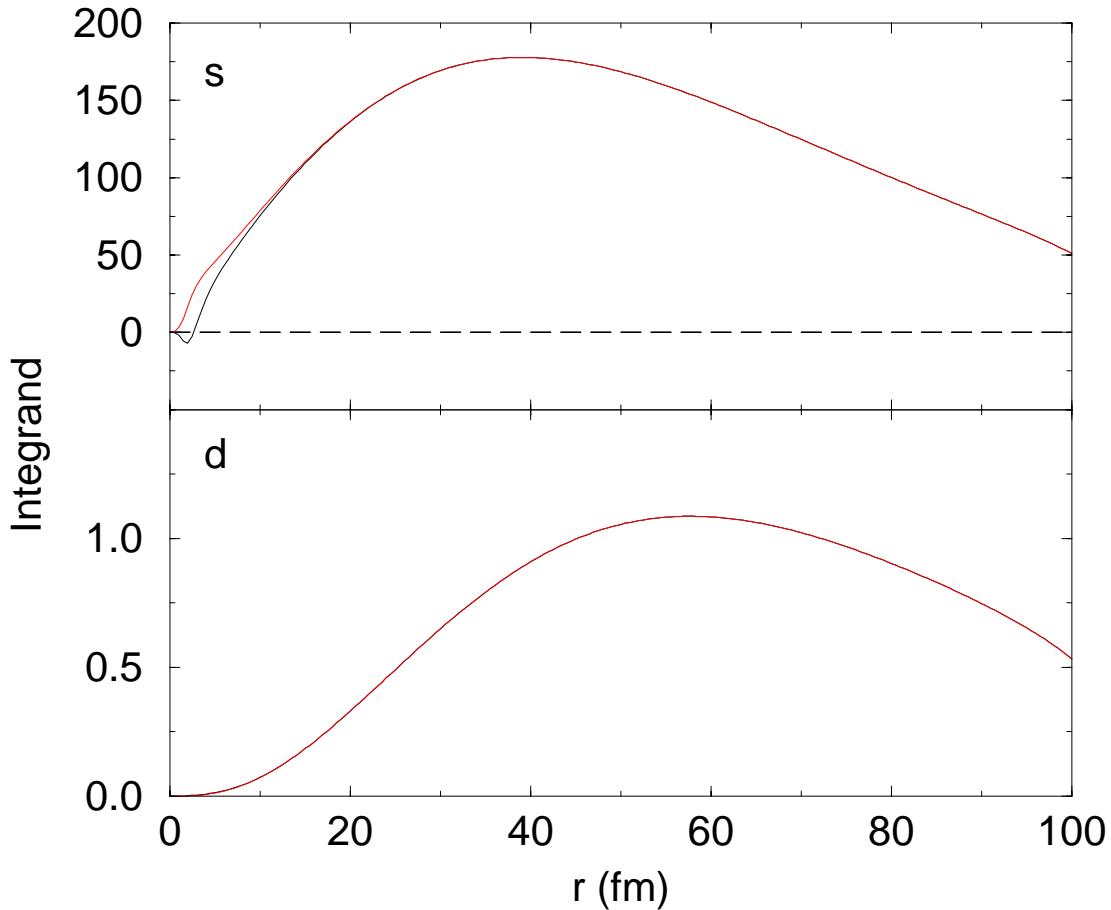
Simple Models



χ^2 per degree of freedom is 0.90 and the S factor at 20keV is 15.3.

χ^2 per degree of freedom is 0.90 and the S factor at 20keV is 20.3. This is 30% higher.

Integrand at threshold



$$I_L = \int_0^\infty dr \ r^2 \ \phi_i(r) \phi_f(r)/k$$

Peak is at 40 fm in the s-wave. Integrand extends well beyond 100 fm.

One Body Operators

$$\mathcal{F} = \int dr dr' a^\dagger(r) a(r') F(r, r')$$

$$\langle \psi_A^n | \mathcal{F} | \psi_A^{n'} \rangle = \int dr dr' F(r, r') \langle \psi_A^n | a^\dagger(r) a(r') | \psi_A^{n'} \rangle$$

$$\begin{aligned} \rho(n, r, n', r') &= \sum_m \langle \psi_A^n | a^\dagger(r) | \Psi_{A-1}^m \rangle \langle \Psi_{A-1}^m | a(r') | \psi_A^{n'} \rangle \\ &= \sum_m \phi_m^{*n}(r) \phi_m^{n'}(r') \end{aligned}$$

- one-body overlap function
 - $\phi_m^{n'}(r') = \langle \Psi_{A-1}^m | a(r') | \psi_A^{n'} \rangle$
- spectroscopic amplitude
 - Spectroscopic factor: $S_m = \int dr |\phi_m^n(r)|^2$

${}^7\text{Be}(\text{p},\gamma){}^8\text{B}$

$F(r, r') \implies \delta(r - r') \exp(i \vec{k} \cdot \vec{r})$

$|\psi_A^{n'}\rangle \implies$ the p ${}^7\text{Be}$ scattering state.

$\langle \psi_A^n | \implies {}^8\text{B}$ final state.

$\langle \Psi_{A-1}^m | \implies$ complete set of ${}^7\text{Be}$ states

$\rho(n, r, n', r') \implies \phi_0^{*n}(r) \phi_0^{n'}(r')$

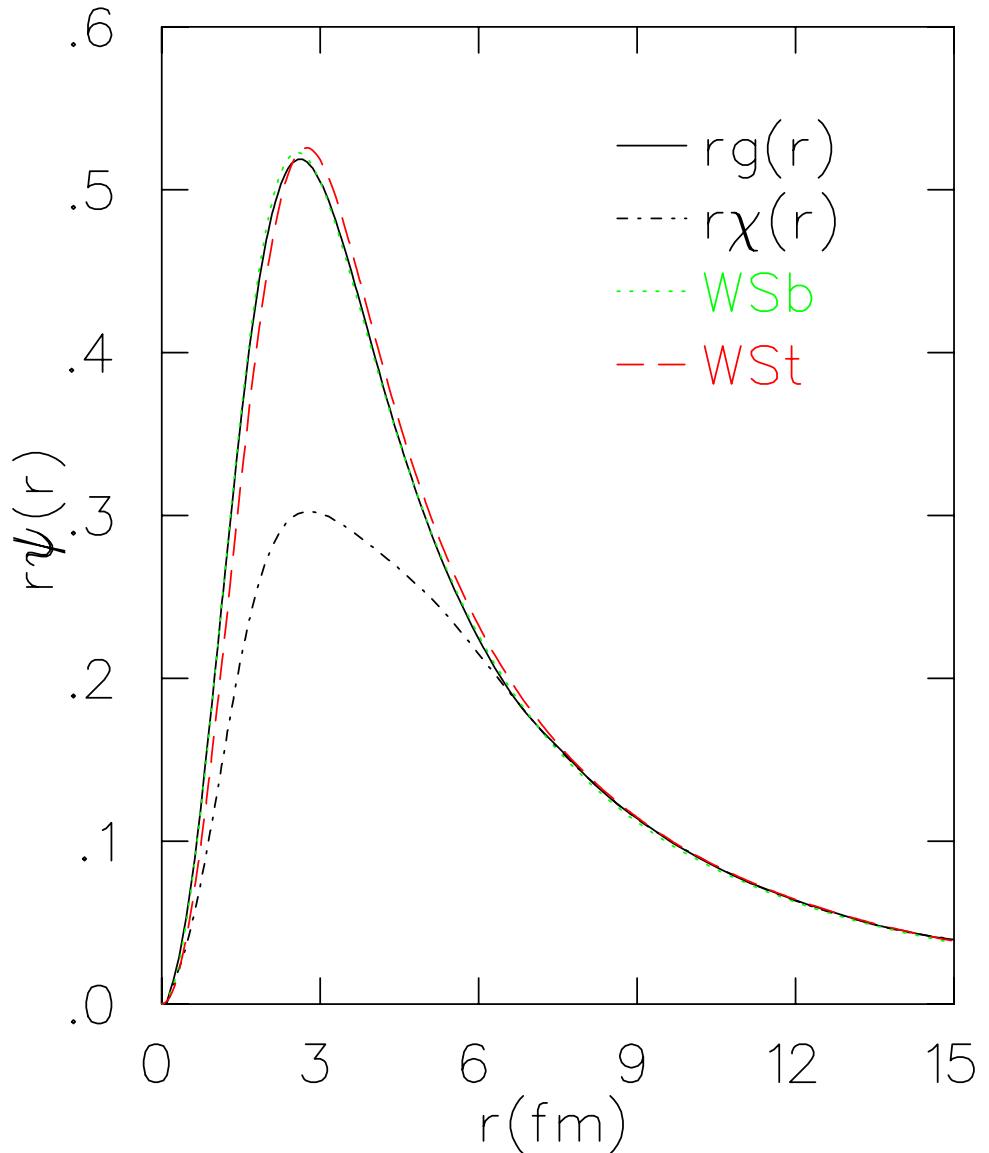
$\phi_0^{*n}(r) = \langle \Psi_{A-1}^m | a(r) | \psi_A^n \rangle$ spectroscopic amplitude for the ${}^8\text{B}$ ground state.

$\phi_0^{n'}(r') = \langle \Psi_{A-1}^m | a(r') | \psi_A^{n'} \rangle$ optical-model wave function for p ${}^7\text{Be}$ scattering state.

Spectroscopic factor: $S_m = \int dr |\phi_m^n(r)|^2$

$\mathcal{M} = \int dr \phi_0^{*n}(r) \exp(i \vec{k} \cdot \vec{r}) \phi_0^{n'}(r')$

The Spectroscopic amplitude



Spectroscopic amplitude \iff one-body wave function

Why the good agreement?

- potential not local in cluster model approach
 - related to Feshbach projection formalism
 - particle approach
- potential approximately local in nuclear mean field
 - particle-hole approach
 - mass operator becomes potential

$$\bar{\phi}_n(r) = \int dr' \mathcal{N}(r, r')^{-1/2} \phi_n(r')$$

$$\begin{aligned}\mathcal{N}(r, r') &= \langle \psi_A | a(r) a^\dagger(r') | \psi_A^{m'} \rangle \\ &= \sum_n \phi^{*n}(r) \phi^n(r')\end{aligned}$$

Equations

- Particle-Hole Approach

$$\begin{aligned} & \left((E_m^{A+1} - E_0^A) + \frac{\hbar^2}{2m} \nabla_r^2 \right) \phi_m^{A+1}(r) \\ & - \int dr' \Sigma(r, r'; E_m) \phi_m^{A+1}(r') = 0 , \end{aligned}$$

- Particle Only Approach

$$\begin{aligned} E_m^{A+1} \phi_m^{A+1}(r) &= \int dr' dr'' \langle \Psi_A | a(r) \\ &\quad \left(H + HQ \frac{1}{E_m^{A+1} - QHQ} QH \right) \\ &\quad a^\dagger(r') | \Psi_A \rangle \mathcal{N}^A(r', r'')^{-1} \phi_m^{A+1}(r'') \end{aligned}$$

$$\begin{aligned} P &= \int dr dr' a^\dagger(r) | \Psi_A \rangle \mathcal{N}^A(r, r')^{-1} \langle \Psi_A | a(r') \\ Q &= 1 - P \end{aligned}$$

Particle-Only Approach

- $\phi(r)$

$$E_m^{A+1} \phi_m^{A+1}(r) = \int dr' dr'' \langle \Psi_A | a(r) \left(H + HQ \frac{1}{E_m^{A+1} - QHQ} QH \right) a^\dagger(r') | \Psi_A \rangle \mathcal{N}^A(r', r'')^{-1} \phi_m^{A+1}(r'')$$

- $\bar{\phi}(r)$

$$E_m^{A+1} \bar{\phi}_m^{A+1}(r) = \int dr' dr'' dr''' \mathcal{N}^A(r, r')^{-1/2} \langle \Psi_A | a(r') \left(H + HQ \frac{1}{E_m^{A+1} - QHQ} QH \right) a^\dagger(r'') | \Psi_A \rangle \mathcal{N}^A(r'', r''')^{-1/2} \bar{\phi}_m^{A+1}(r''')$$

Equations of Motion

- Particle (or hole) approach
 - cluster models
 - Feshbach projection formalism
 - ϕ are not complete
 - $\bar{\phi}$ are (almost) complete
- Particle-hole approach
 - nuclear mean field
 - mass operator
 - Mahaux and Sartor
 - ϕ complete

Proton Emission

$$[H(E) - E]\phi(\mathbf{r}, E) = 0$$

$$[H(E) - E] \frac{\partial \phi(\mathbf{r}, E)}{\partial E} = \left[1 - \frac{\partial H(E)}{\partial E} \right] \phi(\mathbf{r}, E).$$

Multiply by $\phi^*(\mathbf{r}, E)$ and integrate radius r_l .

$$\begin{aligned} & -\frac{\hbar^2}{2m_k(r_l)} \left[\phi_r^*(r_l, E) \frac{\partial \phi'_r(r_l, E)}{\partial E} - \phi'^*_r(r_l, E) \frac{\partial \phi_r(r_l, E)}{\partial E} \right] \\ &= \int_0^{r_l} dr \phi_r^*(r, E) \left[1 - \frac{\partial H_r(E)}{\partial E} \right] \phi_r(r, E), \end{aligned}$$

$$\phi_r(r, E) \underset{r > r_0}{\approx} \cos \delta(E) F(kr) + \sin \delta(E) G(kr).$$

$$\frac{\partial \phi_r(r, E)}{\partial E} \approx \frac{d\delta(E)}{dE} [-\sin \delta(E) F(kr) + \cos \delta(E) G(kr)].$$

Proton Emission

At the resonance energy $d\delta(E_0)/dE = 2/\Gamma_0$,

$$\Gamma_0 \approx \frac{\hbar v_0}{\int_0^{r_t} dr \phi_r^*(r, E_0) \left[1 - \frac{\partial H_r(E_0)}{\partial E} \right] \phi_r(r, E_0)},$$

$$S_0 = \left[\int_0^\infty dr \phi_r^*(r) \phi_r(r) \right] / \left\{ \int_0^\infty dr \phi_r^*(r) \left[1 - \frac{\partial H_r(E)}{\partial E} \right] \phi_r(r) \right\}$$

$$\Gamma_0 \approx \frac{S_0 \hbar v_0}{\int_0^{r_t} dr |\phi_{r0}^{\text{true}}(r)|^2},$$

Proton Emission

$$[H(E) - E]\phi(\mathbf{r}, E) = 0$$

$$[H(E) - E] \frac{\partial \phi(\mathbf{r}, E)}{\partial E} = \left[1 - \frac{\partial H(E)}{\partial E} \right] \phi(\mathbf{r}, E).$$

$$\begin{aligned} S_0 &= \left[\int_0^\infty dr \phi_r^*(r) \phi_r(r) \right] / \\ &\quad \left\{ \int_0^\infty dr \phi_r^*(r) \left[1 - \frac{\partial H_r(E)}{\partial E} \right] \phi_r(r) \right\} \end{aligned}$$

$$\Gamma_0 \approx \frac{S_0 \hbar v_0}{\int_0^{r_t} dr |\phi_{r0}^{\text{true}}(r)|^2},$$

One-body Approaches

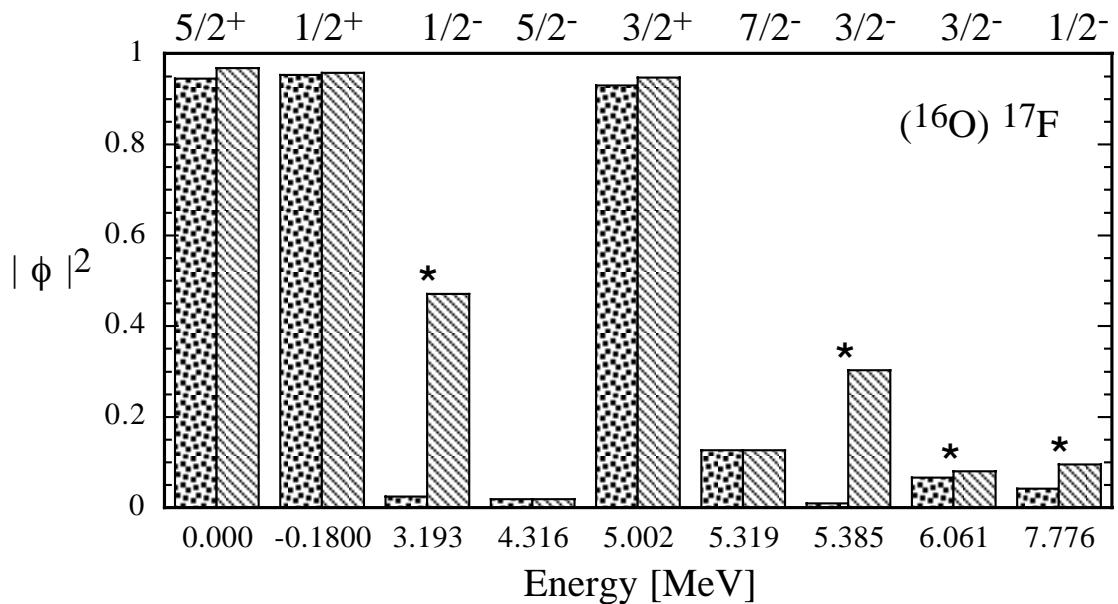
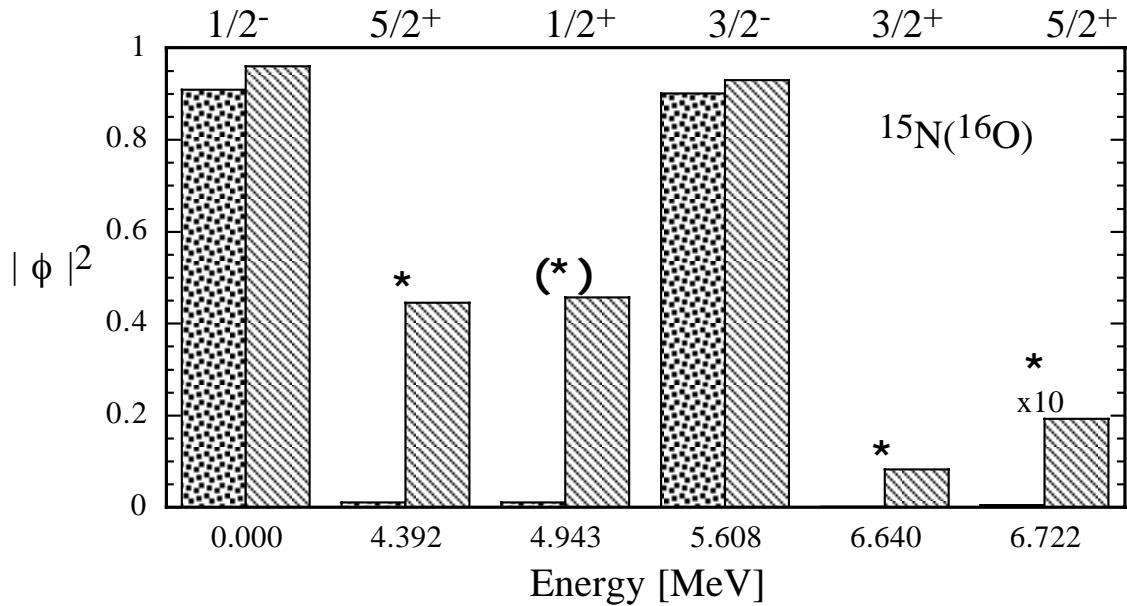
- $\phi(r)$

$$E_m^{A+1} \phi_m^{A+1}(r) = \int dr' dr'' \langle \Psi_A | a(r) \left(H + HQ \frac{1}{E_m^{A+1} - QHQ} QH \right) a^\dagger(r') | \Psi_A \rangle \mathcal{N}^A(r', r'')^{-1} \phi_m^{A+1}(r'')$$

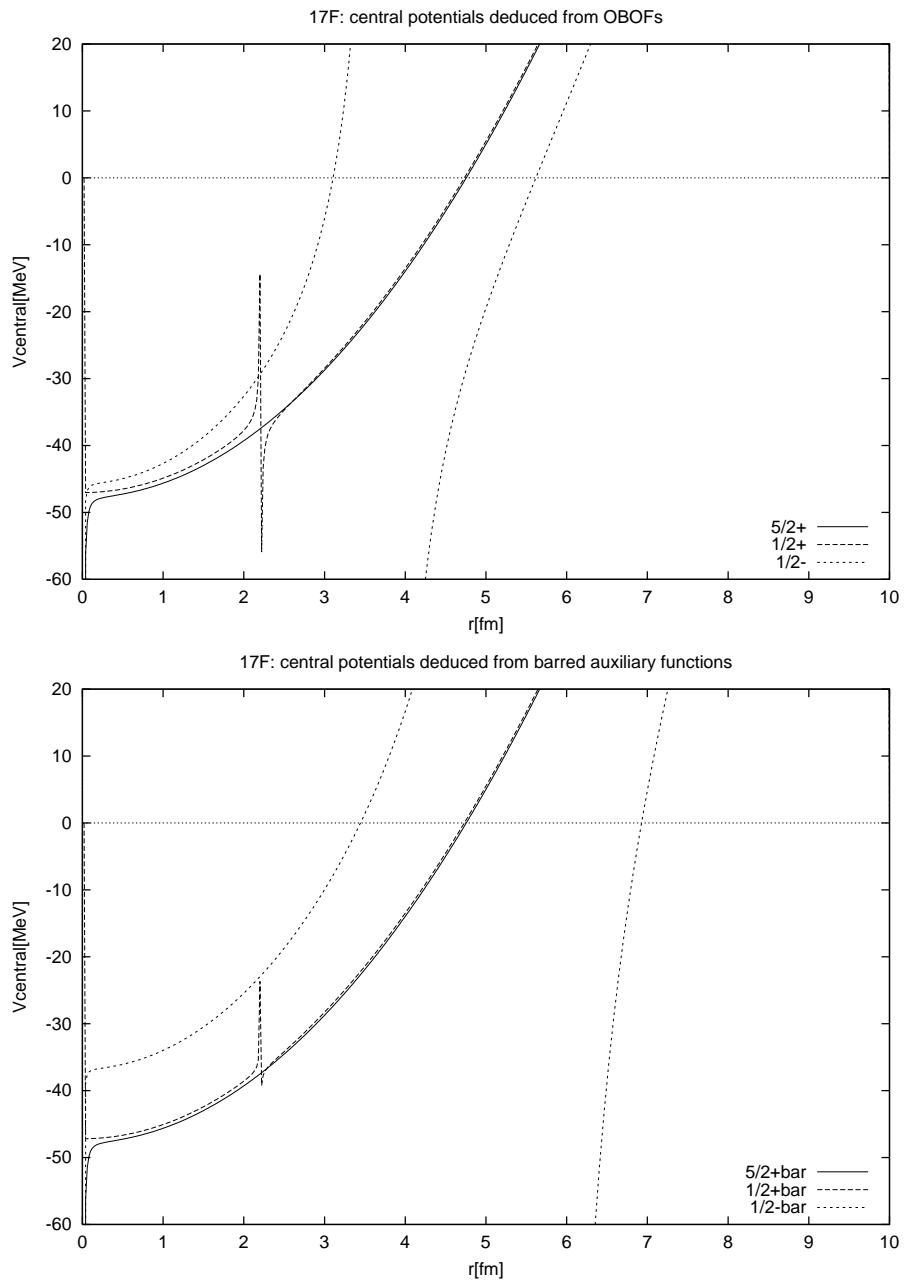
- $\bar{\phi}(r)$

$$E_m^{A+1} \bar{\phi}_m^{A+1}(r) = \int dr' dr'' dr''' \mathcal{N}^A(r, r')^{-1/2} \langle \Psi_A | a(r') \left(H + HQ \frac{1}{E_m^{A+1} - QHQ} QH \right) a^\dagger(r'') | \Psi_A \rangle \mathcal{N}^A(r'', r''')^{-1/2} \bar{\phi}_m^{A+1}(r''')$$

Spectra ^{16}O



Is the potential local?



Conclusions

- Ubiquitousness of one-body approximations
 - radiative capture
 - proton emission
 - etc
- Is the spectroscopic factor, S , an observable?
 - In proton emission it is not!
- Transfer of data from one reaction to another very non-trivial!

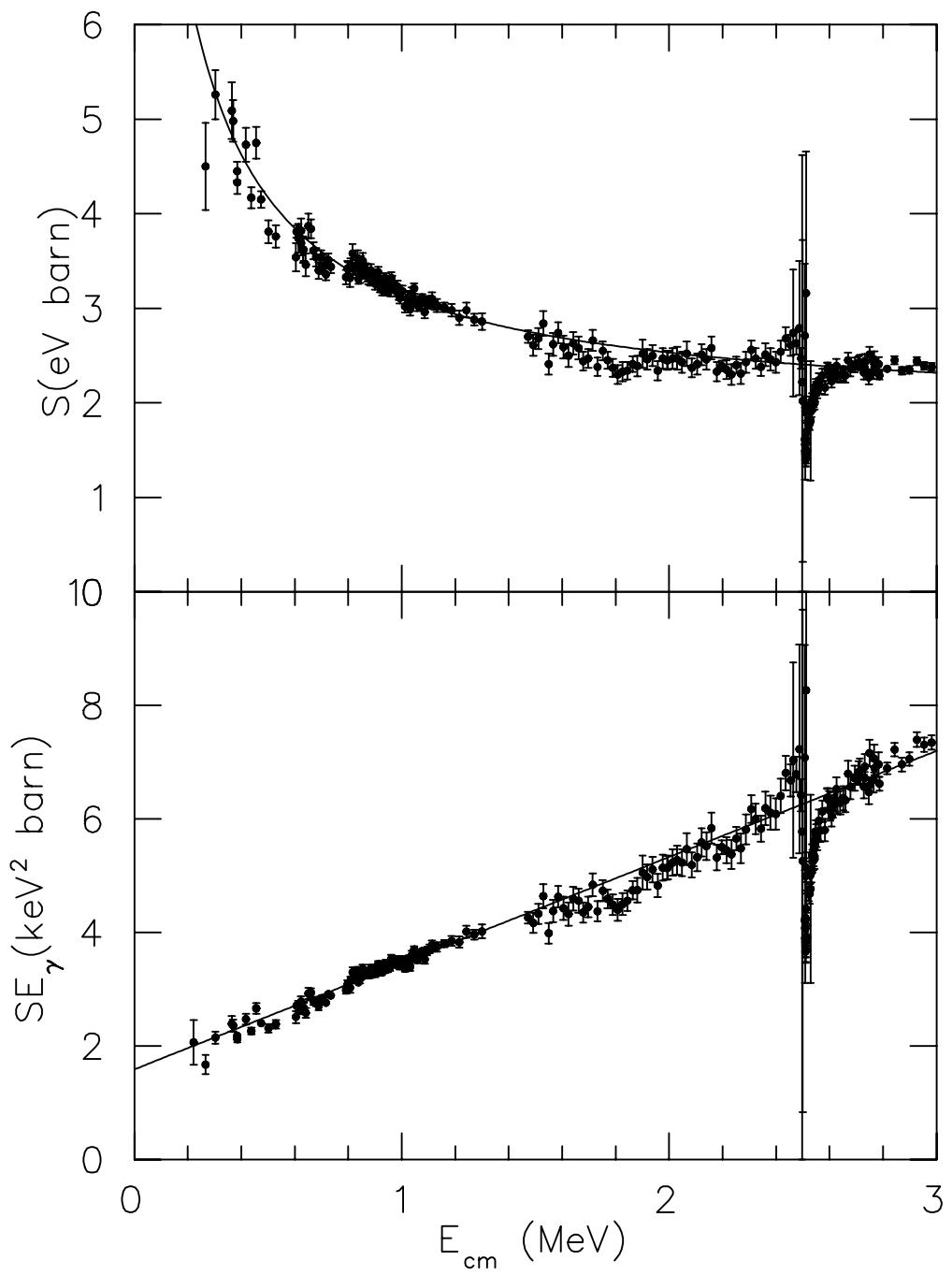
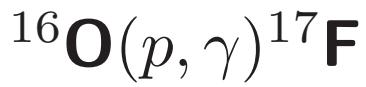
Poles and Padés

Williams and Koonin do an explicit expansion about zero energy for S_{17} and give the first and second logarithmic derivatives as -2.350 MeV^{-1} and 28.3 MeV^{-2} respectively. Using these in a (1,1) Padé approximate gives

$$0.675 \frac{0.206 + E_{\text{c.m.}}}{0.139 + E_{\text{c.m.}}}$$

where $E_{\text{c.m.}}$ is the center-of-mass energy. This Padé approximant has a pole at 139 keV which is very close to the bound state energy of 136 keV that was used in that calculation. Thus we see that in the region of the threshold the cross-section is dominated by the pole.

$$E_\gamma = E_{\text{c.m.}} + E_b$$



Surrogate Warning

